Calculus 1 Name________________________

Review - Implicit Diff. and Related Rates (2) Block ______ Date ________________

1) Find the equation of the tangent line at the given point. $5x^2y^3 + 5x - 6y = 18xy$ $(3,1)$

$$\boxed{10xy^3} + 5x^2 \cdot 3y^2 \frac{dy}{dx} \boxed{+5} - 6\frac{dy}{dx} = 18y + \boxed{18x \cdot \frac{dy}{dx}}$$

$$15x^2y^2\frac{dy}{dx} - 6\frac{dy}{dx} - 18x\frac{dy}{dx} = 18y - 10xy^3 - 5$$

$$\frac{dy}{dx}\left(15x^2y^2 - 6 - 18x\right) = 18y - 10xy^3 - 5$$

$$\frac{dy}{dx} = \frac{18y - 10xy^3 - 5}{15x^2y^2 - 6 - 18x} = \frac{18 \cdot 1 - 10 \cdot 3 \cdot 1^3 - 5}{15 \cdot 3^2 \cdot 1^2 - 6 - 18 \cdot 3} = \frac{-17}{75}$$

$y = mx + b$

$1 = \frac{-17}{75}(3) + b$

$b = 42/25$

$$y = \frac{-17}{75}x + \frac{42}{25}$$

2) Find the derivative of $4x + 3x^2y^4 = 30$.

$$4 + 6xy^4 + 3x^2 \cdot 4y^3 \frac{dy}{dx} = 0$$

$$12x^2y^3 \frac{dy}{dx} = -4 - 6xy^4$$

$$\frac{dy}{dx} = \frac{-4 - 6xy^4}{12x^2y^3}$$

$$\frac{dy}{dx} = \frac{-2 - 3xy^4}{6x^2y^3}$$

2) Find the derivative of $4x+3x^2y^4=30$.

3) Find the derivative of $4\sin(3x^3y)+5xy-7y^4=35$.

$$4\cos(3x^3y)\cdot\left(9x^2y+3x^3\frac{dy}{dx}\right)+5y+5x\frac{dy}{dx}-28y^3\frac{dy}{dx}=0$$

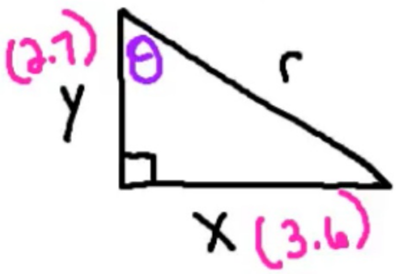
$$36x^2y\cos(3x^3y)+\underline{12x^3\cos(3x^3y)\frac{dy}{dx}}+5y+\underline{5x\frac{dy}{dx}}-\underline{28y^3\frac{dy}{dx}}$$

$$12x^3\cos(3x^3y)\frac{dy}{dx}+5x\frac{dy}{dx}-28y^3\frac{dy}{dx}=-36x^2y\cos(3x^3y)-5y$$

$$\frac{dy}{dx}\left(12x^3\cos(3x^3y)+5x-28y^3\right)=-36x^2y\cos(3x^3y)-5y$$

$$\frac{dy}{dx}=\frac{-36x^2y\cos(3x^3y)-5y}{12x^3\cos(3x^3y)+5x-28y^3}$$

4) A right triangle has legs with lengths of 2.7ft and 3.6ft. The longer leg is increasing at a rate 7ft/min and the shorter side is increasing at a rate of 6.5 ft/min. How fast is the hypotenuse changing at this instant? How fast is the angle between the shorter side and the hypotenuse changing?



a) $x^2+y^2=r^2$

$3.6^2+2.7^2=r^2$

$r=4.5$

$2x\frac{dx}{dt}+2y\frac{dy}{dt}=2r\frac{dr}{dt}$

$2\cdot 3.6\cdot 7+2\cdot 2.7\cdot 6.5=2\cdot 4.5\cdot\frac{dr}{dt}$

$\frac{dr}{dt}=9.5$ ft/min

$\frac{dx}{dt}=7$

$\frac{dy}{dt}=6.5$

b) $\tan\theta=\frac{x}{y}$

$\tan^{-1}\theta=\frac{3.6}{2.7}$

$\tan\theta=x\cdot y^{-1}$

$\frac{d\theta}{dt}=-.222$ rd

$\sec^2\theta\frac{d\theta}{dt}=\frac{dx}{dt}\cdot y^{-1}+x\cdot -1y^{-2}\frac{dy}{dt}$

$\sec^2(.927)\frac{d\theta}{dt}=7\cdot 2.7^{-1}+3.6\cdot -1\cdot 2.7^{-2}\cdot 6.5$

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5) Sand is being poured at a rate of 7.6 cubic centimeters per second and is forming an conical pile. The height of the pile is always a third of the radius. Determine how fast the height is changing at the instant the height is 5 cm. Determine the rate of change of the radius at that same instant.

$\boxed{h = \frac{1}{3} r}$

$\boxed{3h = r}$

$3 \cdot 5 = r$

$15 = r$

$\frac{dV}{dt} = 7.6$

$h = 5$

a) $V = \frac{\pi}{3} r^2 h$

$V = \frac{\pi}{3}(3h)^2 \cdot h$

$V = 3\pi h^3$

$\frac{dV}{dt} = 9\pi h^2 \frac{dh}{dt}$

$7.6 = 9\pi(5)^2 \frac{dh}{dt}$

$\frac{dh}{dt} = .011 \text{ cm/sec}$

b) $V = \frac{\pi}{3} r^2 h$

$V = \frac{\pi}{3} r^2 \cdot \frac{1}{3} r$

$V = \frac{\pi}{9} r^3$

$\frac{dV}{dt} = \frac{\pi}{3} r^2 \cdot \frac{dr}{dt}$

$7.6 = \frac{\pi}{3} (15)^2 \cdot \frac{dr}{dt}$

$\frac{dr}{dt} = .032 \text{ cm/sec}$

6) A perfect cube has side length of 14 in and the sides are decreasing at a rate of .5 in per second. What is the rate of change of the volume of this cube? What is the rate of change of the surface area?

a) $V = s^3$

$\frac{dV}{dt} = 3s^2 \frac{ds}{dt}$

$\frac{dV}{dt} = 3 \cdot 14^2 \cdot -.5$

$= -294 \text{ in}^3/\text{sec}$

b) $SA = 6s^2$

$\frac{dSA}{dt} = 12s \frac{ds}{dt}$

$= 12 \cdot 14 \cdot -.5$

$= -84 \text{ in}^2/\text{sec}$